Game Theory

Lecture 00: Course Overview

Game Theory-- Intro

- **Classical Definition**: Game theory is "the study of mathematical models of conflict and cooperation between intelligent **rational** decision-makers."

- **More Modern Engineering Definition**: Game theory is about interacting agents whose actions affect one another's state of well-being, so that actions taken by one agent has spillover effects on other agents in the system. These externalities create a complex environment of action-reaction. What we are interested is the dynamics of the system (game) and what kind of equilibria the system will tend to.

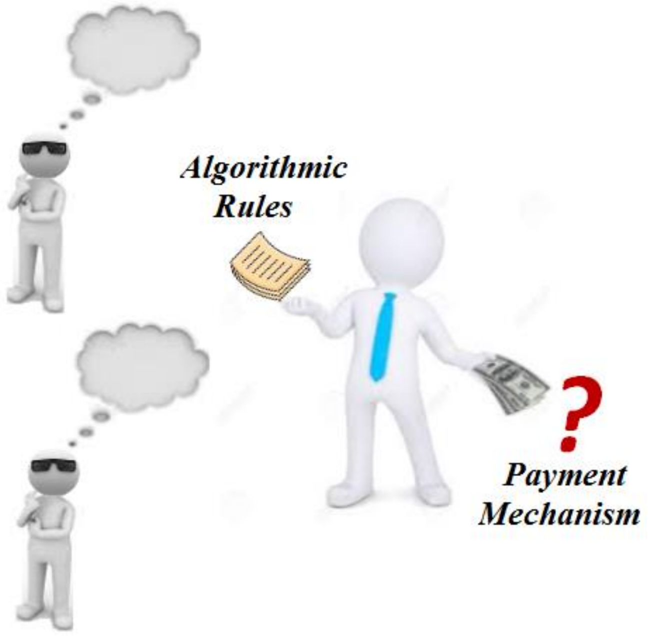
- Model: n agents , each chooses some $x_i \in \mathcal{R}$,and has a utility function $u_i(\mathbf{x})$, $\mathbf{x} \in \mathcal{R}^n$, or equivalently,

$$u_i(x_i \, ; x_{-i} = \; x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$$

  - ★ What are the possible outcome?
  - ★ Steady-state, stable operating point, characteristics?
  - ★ How do you get there (learning dynamics, computation of equilibrium)?

Descriptive Agenda

- **Assumptions**
  - The players are "self-interested" "rational" agents
  - Utility functions are part of the modeling process (not a design choice)
  - Distributed algorithmic rules can only be "suggested" to the players not (forcibly) "dictated"
- **Main Challenge**
  - The tension between the private goals of the players and the goal of the designer who is concerned with social welfare
- **Objective**
  - To influence the behavior of the players to realize the social goal
    - By Mechanism Design: e.g., social norms or conventions, monetary payments, reputation mechanisms, etc.



Engineering Agenda

- Recently, game theory has emerged as a valuable tool for controlling or prescribing behavior in distributed engineered systems.
- The rationale for this new perspective stems from the parallels between the underlying decision making architectures in both societal systems and distributed engineered systems.
  - In particular, both settings involve an interconnection of decision making elements whose collective behavior depends on a compilation of local decisions that are based on partial information about each other and the state of the world.

Engineering Agenda

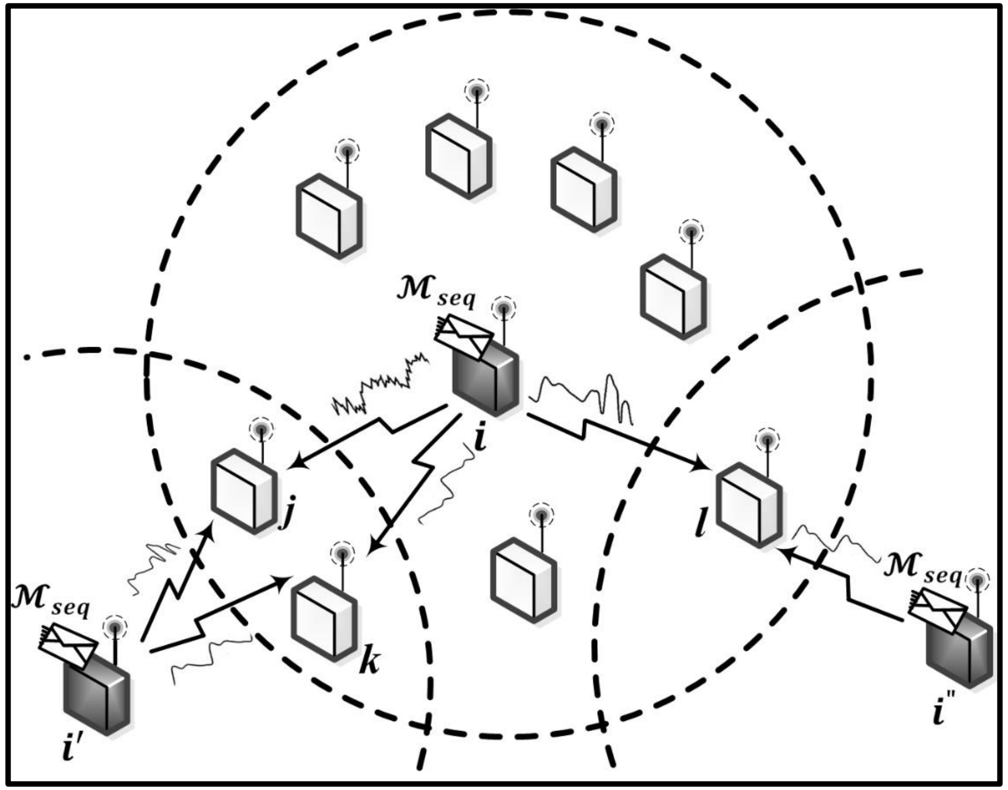
- **Assumptions**
  - The players are not necessarily rational agents
  - Utility functions are considered as design choices
  - Unlike mechanism design, the agents in the engineered agenda are programmable components. Accordingly, there is no concern that agents are not truthful in reporting information or obedient in executing instructions.

- **Main Challenge**
  - To synthesize distributed algorithmic rules for reaching efficient game-theoretic equilibria

- **Justifications for a Game-Theoretic Formulation**
  - Coupling between the agents' utility functions
  - A systematic attempt to distributed performance optimization
  - Information sharing induces high signaling overhead
  - Guarantees some form of consensus between the agents and guides the system towards a stable operating point (consistent with some notion of sub-optimality)

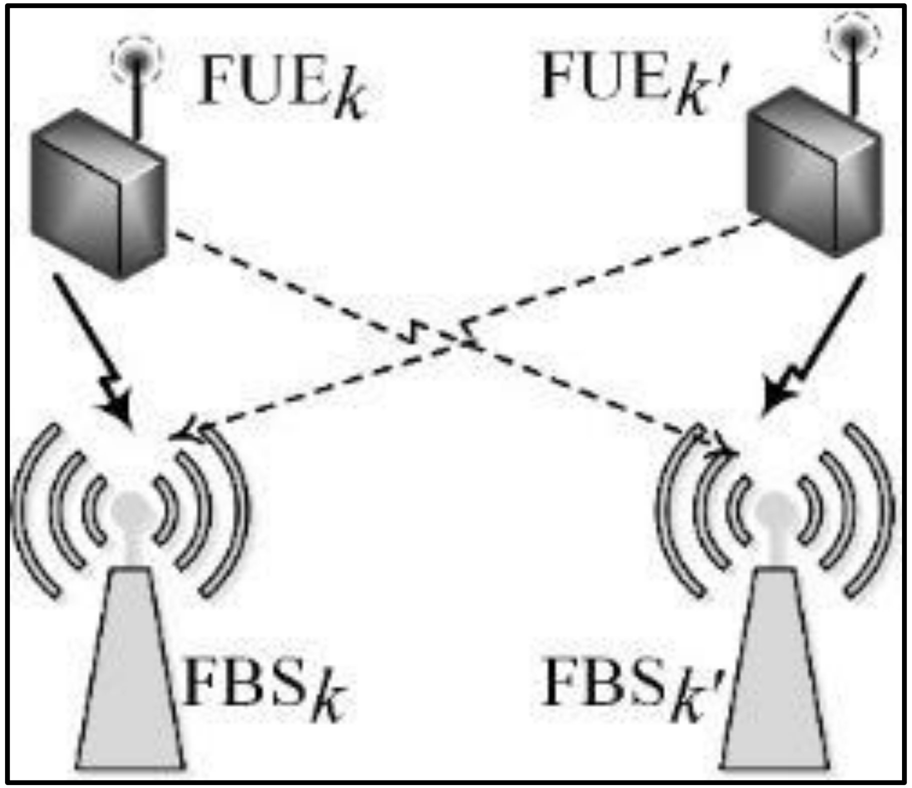
Motivating Scenarios: **coupled distributed optimization**

Participating in a shared service



Players: nodes i, i', i”
Action: forward/drop
Utility: coverageRatio-α(forwardingCost)

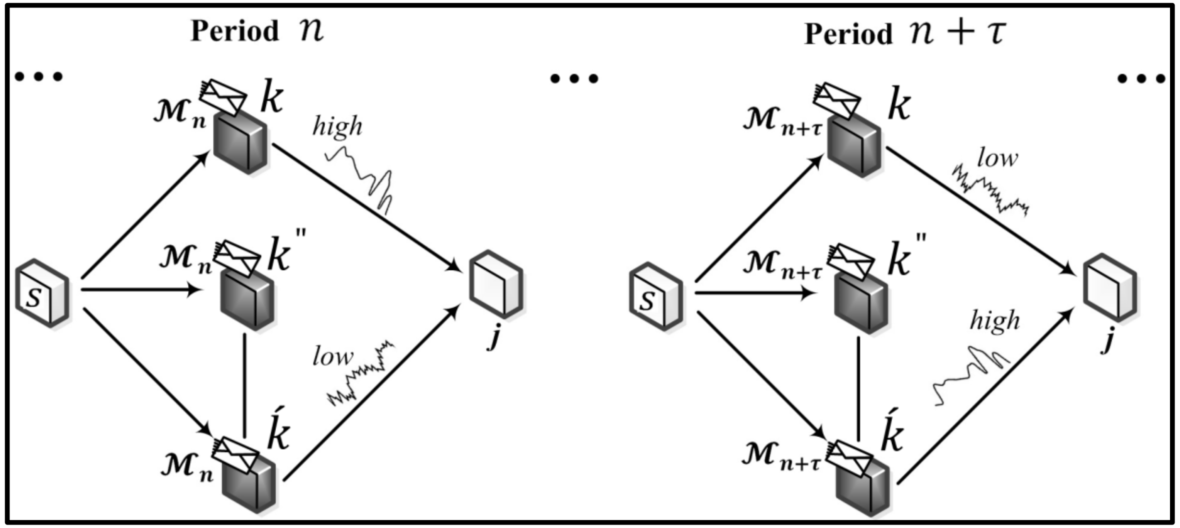
Using a shared resource

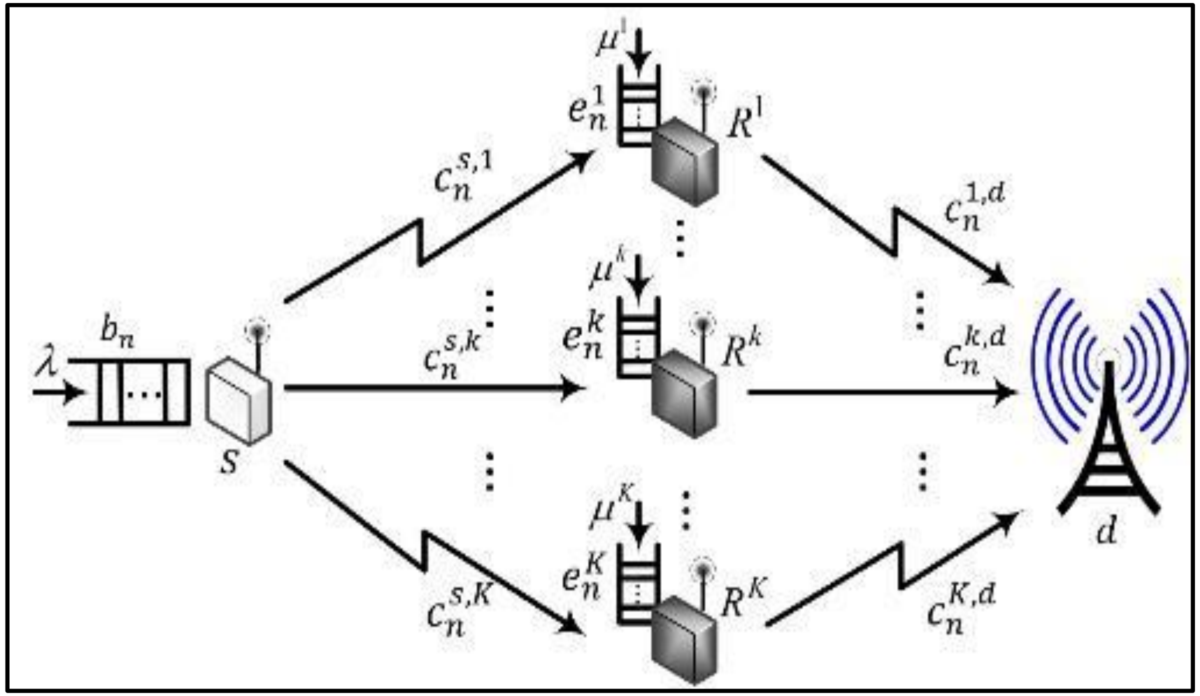


Players: FUE k, k'
Action: power level
Utility: Shannon rate (physical throughput)

More Advanced Scenarios:

coupled distributed “stochastic” optimization





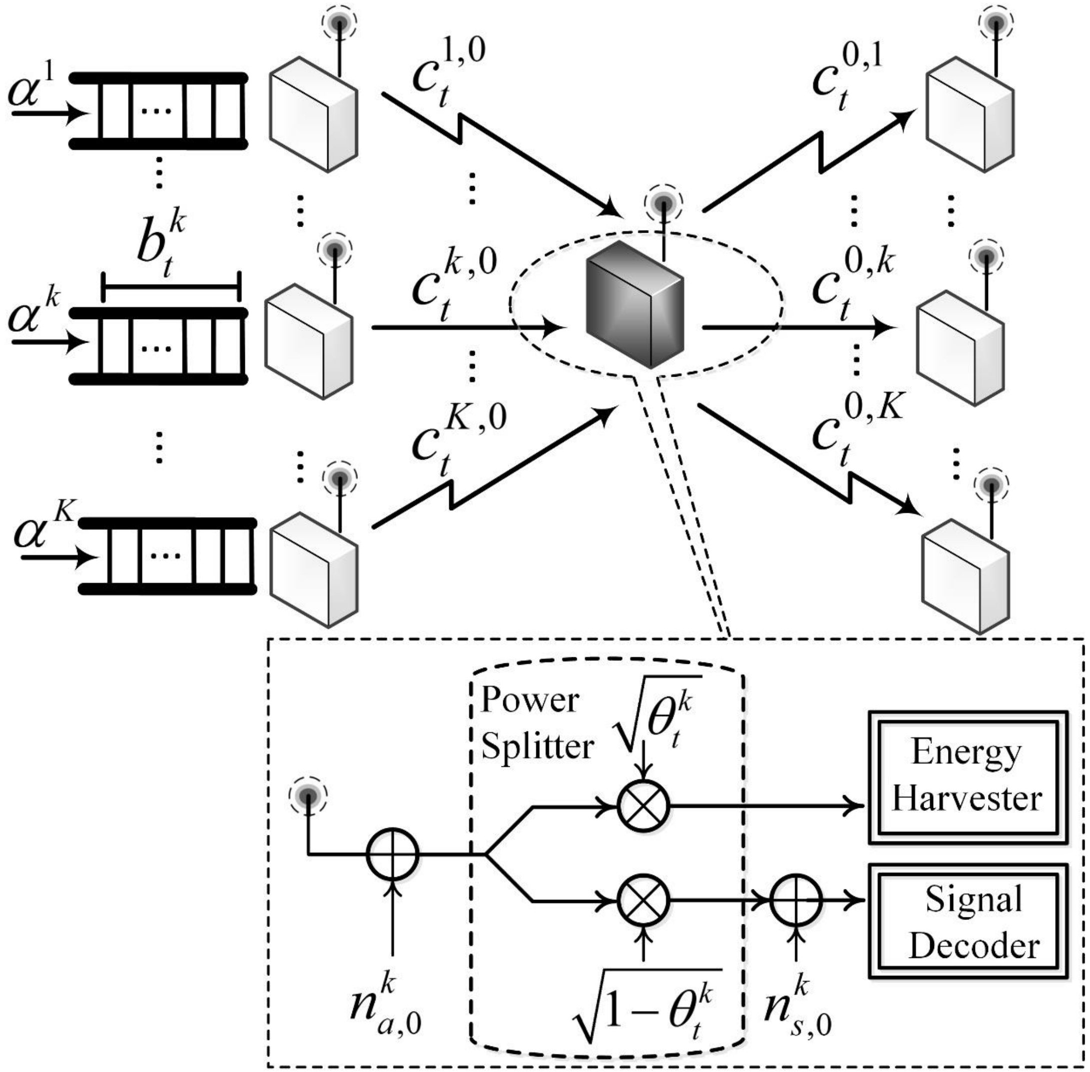
Players: nodes k, k‘
Action: forward/drop
Utility: coverageRatio-α(forwardingCost)

Slowly time-varying
(due to slow fading channels)

Players: Relays 1, …, k,… K
Action: power level
Common Utility:

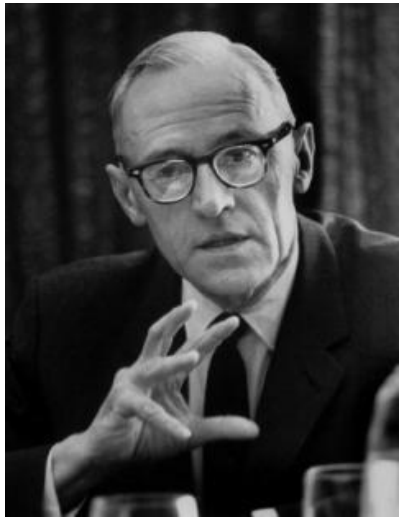
Average Delay of the Source Buffer

Motivating Scenarios:
Faithful implementation of Distributed Algorithmic Rules

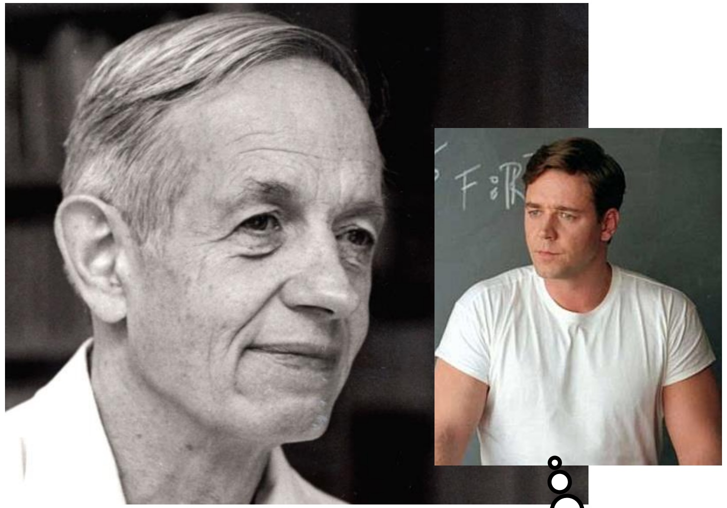


- An RF Energy-Harvesting Relay
- Multiple Users with Bursty Traffic Arrival
- Orthogonal Fading Channels
- Simultaneous Wireless Information and Power Transmission
- Decode and Forward Relaying Protocol
- Power Splitting between "Decoding" and "Harvesting" Units

Popular Figures in Game theory

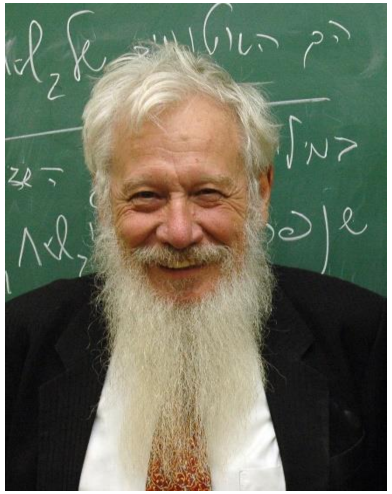



John von Neumann (left) and Oskar Morgenstern (right)
Known for:
Theory of Games and Economic Behavior

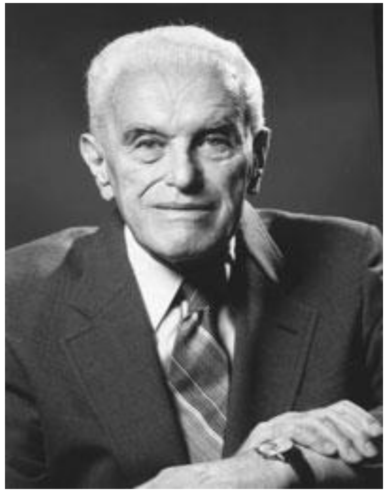


John F. Nash
Known for:
Nash Equilibrium

A Beautiful Mind



Robert Aumann
Known for:
Correlated Equilibrium



John Harsanyi
Known for:
Bayesian Games

Roger Myerson
Known for:
Mechanism Design

A Taxonomy of Games

- **Spatial**
  - Dimensionality of the Strategy, Player, or State Spaces
- **Temporal**
  - Simultaneous (one-shot, static, normal)
  - Sequential (multi-stage/extensive)
- **Informational**
  - Perfect vs. Imperfect Monitoring
  - Perfect vs. Imperfect Recall
  - Complete vs. Incomplete Information
- **Player Cooperation**
  - Non-cooperative Vs. Coalitional
- **Uncertainty**
  - Stochastic vs. Deterministic

Course Outline

- **Part I. Fundamentals of Game Theory**
  - Normal-Form Games, Utility Functions, Strategies (pure, mixed), Best Response, etc.
  - Famous Solution Concepts
    - Iterated Elimination of Strictly Dominated Strategies (IESDS),
    - Rationalizability (Iterated Elimination of Never-a-Best-Response Strategies)
    - Nash Equilibrium (NE)
  - Solution Concepts Beyond NE
    - Correlated Equilibrium
    - minimax strategies
    - minimax regret, etc.
  - Computational Aspects of Game-Theoretic Equilibria
    - Mathematical programs and equilibrium computation algorithms
    - computational complexity, etc.
  - Special Games: weakly acyclic, potential, congestion, supermodular, graphical, etc.

Course Outline

- **Part I. Fundamentals of Game Theory**
  - Extensive-Form Games
    - Game Trees
    - Perfect Information Games
    - Subgame Perfect Equilibrium, Backward Induction
    - Imperfect Information and Imperfect Recall Games, Information Sets, etc.
    - Behavioral and Mixed Strategies
  - Repeated Games
    - Infinite Horizon Repeated Games, Cumulative Payoffs, etc.
    - Tit-for-Tat and Grim-Trigger Strategies, etc.
    - Folk-Theorems
  - Stochastic Games
  - Bayesian Games
  - Coalitional Games

Course Outline

- **Part II. Learning and Dynamics**
  - Evolutionary Games
    - Evolutionary Stable Sets (ESS)
    - Replicator Dynamics
  - Multi-Agent Learning
    - Bayesian Rational Learning
    - Boundedly-Rational Learning
    - Fictitious Play
    - No-Regret Learning
    - Learning MinMax Strategies in Zero-Sum Games
    - Learning CE Strategies in General-Sum Games
    - Discussion of Convergence

Course Outline

- **Part III. Mechanism Design**
  - Auction basics
  - Second-Price Auctions, Truthful Equilibrium in Dominant Strategies
  - Sponsored Search Auctions (Google AdWords, etc.)
  - First-Price Auctions
  - Maximizing Social Welfare: VCG Mechanism
  - Reputation-Based Incentive Mechanisms

Text-Book References

- "Game Theory" by Drew Fudenberg and Jean Tirole, MIT Press, 1991.
- "An Introduction to Game Theory" by Martin J. Osborne, Oxford University Press, 2003
- "Game Theory: An Introduction" by Steven Tadelis, Princeton University Press, 2013.
- "Game theory" by S. Zamir, M. Maschler, and E. Solan, Cambridge: Cambridge University Press, 2013.
- Y. Shoham and K. Leyton-Brown, Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations. Cambridge, U.K.:Cambridge Univ. Press, 2008.
- "Algorithmic Game Theory," N. Nisan, T. Roughgarden, É Tardos, and V. Vazirani, Eds. Cambridge, U.K.: Cambridge Univ. Press, 2007.

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  - Keep up with announcements!

Grading Scheme